Brilliant Failure to (De)Construct the (Im)Possible Problem

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How do you imagine mathematics?

Pure mathematics is, in its way, the poetry of logical ideas.

- Albert Einstein

Do you believe in God?

Well, I do believe in higher powers...
Doing Mathematics means

• Seeking and studying Patterns

• Finding Multiple Representations

• Using Inductive and Deductive Reasoning

Pure mathematics is, in its way, the poetry of logical ideas.

- Albert Einstein
Where do we integrate purposeful failure?

https://www.youtube.com/watch?v=pobLgR6UV5g
Why *Brilliant Failure* versus Failure?

It is not possible to solve the problem on the same level at which it originated. You need to rise above this problem, up, to the next level to see it from other perspective.

"INSANITY IS DOING THE SAME THING, OVER AND OVER AGAIN, BUT EXPECTING DIFFERENT RESULTS."

- Albert Einstein
7 Bridges of Konigsberg

Problem:

• Start on land or bridge, you choose
• Find path where you cross a bridge only once (no backtracking)
• Cannot walk on water, or “go around” the river

What does your brain do when you hit your first obstacle? Second obstacle?
Brilliant Failure

The “what if...” thought that causes you to go back and approach the problem from a different perspective.
Question prompts to support pair problem solving dialogue around “what if...” thoughts

• What (exactly) are you doing?

• Can you describe ____ precisely?

• Why are we doing _______?

• How does ____ fit into the solution?

• How does ____ help us with _____?

• What will we do with _______ when we obtain it?

I’m stuck

What if...?

What if we blow up a bridge to work with 6 bridges instead of 7?

What if we build another bridge to work with 8 bridges instead of 7?
Study of Patterns

CCSS Math Practice 8: Look for and express regularity in repeated reasoning.

Generalization = Short Cut

CCSS Math Practice 7: Look for and make use of structure.
Problem: Arrange these 6 pencils to create 4 equilateral triangles

Seek your “what if....”
Question prompts to support pair problem solving dialogue around “what if...” thoughts

- What *(exactly)* are you doing?
- Can you describe ____ precisely?
- Why are we doing ________?
- How does _____ fit into the solution?
- How does _____ help us with _____?
- What will we do with _______ when we obtain it?
How do we scaffold *Brilliant Failure* and problem solving dialogues?

**Strategic Clue:**

**CCSS Math Practice 2:** Reason abstractly and quantitatively.
Multiple Representations

CCSS Math Practice 4: Model with mathematics.

Map relationships using such tools as diagrams, two-way tables, graphs, flowcharts, and formulas

CCSS Math Practice 5: Use appropriate tools strategically.

Example of a “tool”: logarithms
Logarithms simplify difficult calculations

A real result of *Brilliant Failure*

Gregoire de Saint Vincent’s “what if....”

In an attempt to perform a quadrature of a rectangular hyperbola Gregoire discovered the natural logarithm

(rectangular hyperbola)
Once we figure out a “what if…” that appears to work, we need to figure out...

https://www.youtube.com/watch?v=RRq71LawQB4
Mathematical Proof
Inductive & Deductive Reasoning

**Mathematical Proof**: Develop a tightly knit *chain of reasoning*, following strict logical rules, that leads inexorably to the *absolute truth*.

**CCSS Math Practice 3**: Construct viable arguments & critique the reasoning of others.

**CCSS Math Practice 6**: Attend to precision.
Can we *prove* that every even whole number greater than 2 can *always* be written as the sum of two primes?

Seeking my *Brilliant Failure*, my “what if…” thought that causes me to go back and approach the problem from a different perspective.
Goldbach Conjecture

In 1742, Christian Goldbach came to the following conclusion:

It seems that every even number greater than 2 can indeed be written as the sum of two primes.

He communicated his idea to Leonhard Euler who regarded the result as trivial. However, the Goldbach Conjecture remains unproven to this day.

Unsolved Problems: A reason for teaching “what if…..”
Role of Brilliant Failure in Doing & Thinking Through Mathematics

**MP 1:** Make sense of problems and persevere in solving them.

**MP 2:** Reason abstractly and quantitatively.

**MP 3:** Construct viable arguments and critique the reasoning of others.

**MP 4:** Model with mathematics.

**MP 5:** Use appropriate tools strategically.

**MP 6:** Attend to precision.

**MP 7:** Look for and make use of structure.

**MP 8:** Look for and express regularity in repeated reasoning.
A Roadmap for Instruction that Promotes Mathematical Thinking & Problem Solving

Use and connect mathematical representations

<table>
<thead>
<tr>
<th>Teacher and Student actions</th>
<th>What are students doing?</th>
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</thead>
<tbody>
<tr>
<td>Selecting task that allow students to decide which representations to use in making sense of the problems.</td>
<td>Using multiple forms of representations to make sense of and understand mathematics.</td>
</tr>
<tr>
<td>Allocating substantial instructional time for students to use, discuss, and make connections among representations.</td>
<td>Describing and justifying their mathematical understanding and reasoning with drawings, diagram and other representations.</td>
</tr>
<tr>
<td>Introducing forms of representations that can be useful to students.</td>
<td>Making choices about which forms of representations to use as tools for solving problems.</td>
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<td>Asking students to make math drawings or use other visual supports to explain and justify their reasoning.</td>
<td>Sketching diagrams to make sense of problems situations.</td>
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<tr>
<td>Focusing students’ attention on the structure or essential features of mathematical ideas that appear, regardless of the representation.</td>
<td>Contextualizing mathematical ideas by connecting them to real-world situations.</td>
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<tr>
<td>Designing ways to elicit and assess students’ abilities to use representations meaningfully to solve problems.</td>
<td>Considering the advantages or suitability of using various representations when solving problems.</td>
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</table>
We use *Brilliant Failure* to (De)Construct the (Im)Possible Problem every day as teachers...

https://www.youtube.com/watch?v=SFnMTHhKdkw
Questions?

“My math teacher keeps asking me questions. You’d think she’d know all that stuff by now.”

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